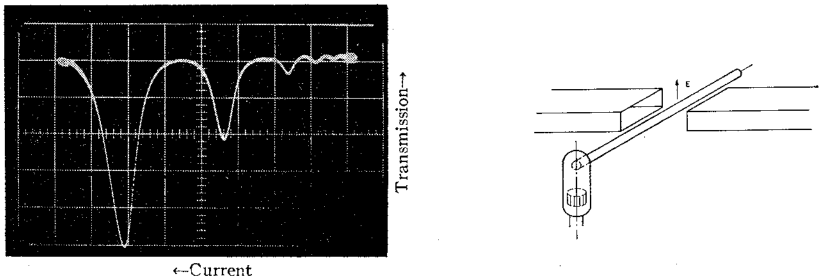
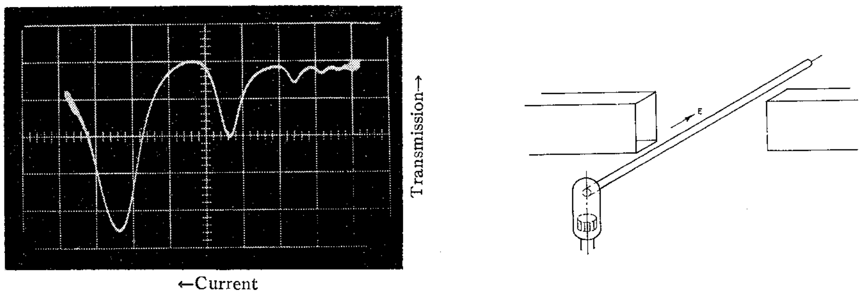
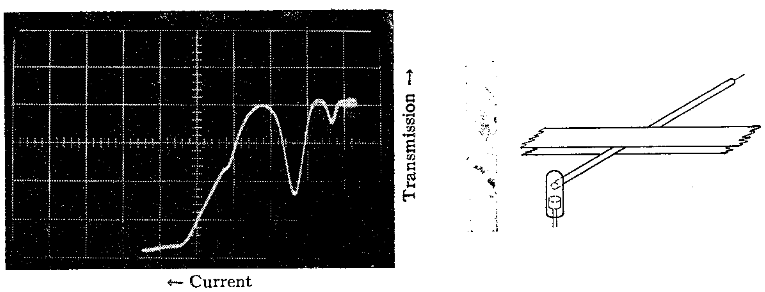


(a)

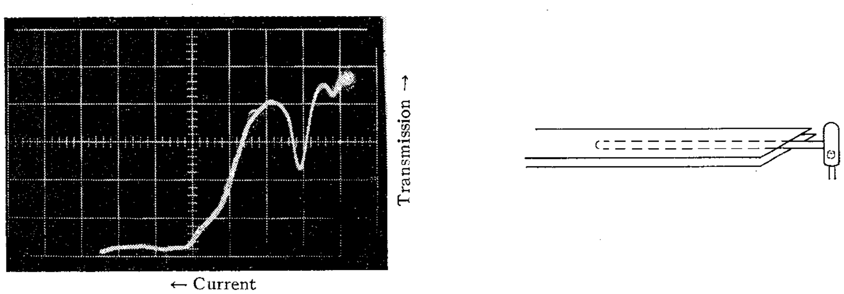


(b)

Fig. 1—(a) E field perpendicular to column. (b) E field parallel to column.



(a)



(b)

Fig. 2—(a) Plasma tube across transmission line. (b) Plasma tube along transmission line.

If now the plasma tube is placed at the center of the transmission line with its axis parallel to the axis of the line as shown in Fig. 2(b) the resulting output from the line again exhibits the Tonks-Dattner resonances. The tube in this case was approximately 1.2 wavelengths long.

The above experiments show that resonances can be excited when the electric vector of the incident EM wave is parallel to the axis of the plasma column and also when the plasma column is placed so that it may not be considered as a localized discontinuity. Since these resonances are not predicted by any theory so far put forward we feel there is a need for variation of the Dattner type experiment and for the performance of new and different experiments so that the conditions under which these oscillations are excited can be more completely determined.

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Proposed Experiment for Eliciting Multiple Resonances from the Ionosphere*

In the theory of Herlofson[1] only one resonance is predicted for a cylindrical plasma column irradiated by an electromagnetic wave having both its direction of propagation and electric field E perpendicular to the axis of the column, a mode which he designates as sagittal. Herlofson treats the problem by solving the wave equation in cylindrical coordinates and then imposing boundary conditions to find the frequency or frequencies for maximum scattering from the column. In his treatment, the modes which involve Bessel functions of order higher than unity have the same resonant frequency as that for the dipolar mode for which the order of the Bessel function is unity. No resonances at all are predicted for the parallel mode of excitation in which E is parallel to the axis of the column. These predictions are contrary to the experimental observations of Dattner[2] and others[3] for the saggital mode and also contrary to the observations reported by Willis and Petroff[4] in which a spectrum of resonances is found for the parallel mode. Experiments by Boley[5] have shown that the sagittal scattering for the higher order resonances is that appropriate for a dipole, that is, his experiments show that the field about the column for the higher-order modes is not quadrupolar or sextupolar.

These multiple resonance experiments under a variety of experimental arrangements suggest that the mode spectrum may be an intrinsic property of a plasma, perhaps of an extended plasma such as the ionosphere. The higher-order resonances occur when the electron density is lower than that required for the principal resonance. The frequencies for the various resonances, f_n (they may also be expressed in terms of electron densities), are given reasonably well by the expression

$$(2n)/(2n+1) = \left[1 - \left(\frac{f_0}{f_n}\right)^2\right]^{1/2} \quad (1)$$

where $n=0$ for the principal resonance and $n=0, 1, 2, 3, \cdots, n$ for the series. f_n may be expressed in terms of f_0 and, for the various values of n we have

$$f_n/f_0 = 1.0, 1.34, 1.67, \text{etc.} \quad (2)$$

An experimental test of this hypothesis would be afforded by taking ionospheric soundings employing a set of discrete frequencies and noting whether reflections from the ionosphere are obtained, at a fixed height, not at one frequency only, but

* Received May 4, 1962.

[1] N. Herlofson, "Plasma resonance in ionospheric irregularities," *Arkiv. f. Physik.*, vol. 3, pp. 247–297; 1951.

[2] A. Dattner, "The plasma resonator," *Ericsson Technics* (Stockholm), vol. 13, pp. 310–350; 1957.

[3] W. D. Hershberger, "Absorption and reflection spectrum of a plasma," *J. Appl. Phys.*, vol. 31, pp. 417–422; February, 1960.

[4] J. Willis and I. Petroff, "Resonances in a cylindrical plasma column," IRE Trans. on Microwave Theory and Techniques, September, 1962.

[5] F. I. Boley, "Scattering of microwave radiation by a plasma column," *Nature*, vol. 182, pp. 790–791; September 20, 1958.

at a set of related frequencies whose ratio are specified in (2). Data of the sort desired may already be available from some laboratory engaged in ionospheric soundings. If not, it is suggested that simultaneous soundings at several frequencies be taken and the results reported. If the experiments gave a positive result, changes obviously would be necessitated in some of our accepted views as to the structure of the ionosphere.

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Stability Criteria for Tunnel-Diode Amplifiers*

A tunnel-diode amplifier is stable when the amplifier network is reduced to an arbitrary single loop and the equation given by the sum of the impedances around this loop equal to zero [$\Sigma Z(P)=0$ where $p=\gamma+j\omega$] has no solution in the right-half plane ($\gamma>0$). This is equivalent to the requirement that the system determinant shall have no zeros in the right-half plane. Several authors have used this criterion.[1,2] To determine analytically whether $\Sigma Z(p)$ has any positive zeros is very laborious if at all possible for many practical amplifier configurations.

A possible approach is to use a contour theorem from the theory of complex functions as formulated by Goldman.[3] "If a function $Z(p)$ is analytic, except for possible poles within a given contour taken in the clockwise direction in the p-plane, then the number of times that this contour into the Z-plane encircles the origin in the Z-plane in a clockwise direction is equal to the number of zeros minus the number of poles of $Z(p)$ inside the contour in the p-plane, each pole and zero being counted in accordance with its multiplicity."

When applying this contour theorem to $\Sigma Z(p)$ for an arbitrary loop in the tunnel diode circuitry, we must have a complete knowledge of the poles of $\Sigma Z(p)$ in the right-half plane and at infinity. Hughes[4] discusses these difficulties and obtains a plot easy to interpret by dividing the original function $Z(p)$ with a new function $Z_2(p)$ with no zeros in the right-half p-plane, the same number of poles as $Z(p)$ in the right-half p-plane and with the same limit at infinity as $Z(p)$ This method is applicable to tunnel-diode amplifiers, but a complete knowledge of the poles in the right-half p-plane is sufficient to interpret the complex plot of $\Sigma Z(p)$ along a chosen contour. Generally we can say that the closer to the negative resistance we choose the loop representing the tunnel diode and amplifier network, the fewer the poles in the right-half p-plane. It is convenient to choose the representation given in Fig. 1 for then the diode is separated from the rest of the network.

$Z'(p)$ includes the cartridge capacitance, bias circuit, matching network, load, etc., and is a passive impedance. The impedance around this loop is given by

$$\sum Z(p) = \frac{\frac{1}{C}}{p - \frac{1}{RC}} + R_s + pL + Z'(p).$$

This function has one pole in the right-half plane at $p=\gamma_p=1/RC$ and a pole at $p=\pm\infty$.

With a contour in the p-plane chosen as the $j\omega$-axis and a semicircle enclosing all poles and zeros in the right-half plane, the ΣZ-plot of a stable amplifier encircles the origin once in a counterclockwise direction as determined by the pole $p=1/RC$. From symmetry it is sufficient to plot $\Sigma Z(j\omega)$ for positive frequencies and only up to the diode cutoff frequency ω_c as the circuit is passive above ω_c. Davidsohn, Hwang and Ober[5] have considered stability criteria from a similar point of view.

Summing up we have the following stability criterion: "*A tunnel-diode amplifier is stable if and only if the sum of the diode impedance and the connected network impedance plotted as a function of frequency encircles the origin once in a counterclockwise direction when the plot is closed with an arbitrary line in the right-half Z-plane between the positive and negative diode cutoff frequency. The diode cartridge capacitance is considered to belong to the network connected to the diode.*"

Fig. 2 shows the application of this criterion. This circuit is one of the simplest possible amplifier circuits and yet it is laborious to investigate the stability analytically.

The graphical display gives a good feeling of how the amplifier stability is influenced by change in diode parameters. This is especially true when the amplifier circuit is more complex than given in the example so that the minimum distance between the origin and the plotted function occurs for frequencies different from the amplifier center frequency.

When an amplifier configuration connected to an ideal transmission line or a load resistance R_0 is determined to be stable, it is of interest to determine which mismatch is permissible at the input without upsetting the stability. To do this we reduce the amplifier network to the loop nearest to the transmission line connection (Fig. 3). Z_{ampl} includes the tunnel diode and matching network Z_{load} is the transmission line impedance with mismatches from circulators, stabilizing networks, etc. A condition for stability is that $Z_{ampl}(p)+Z_{load}(p)=0$

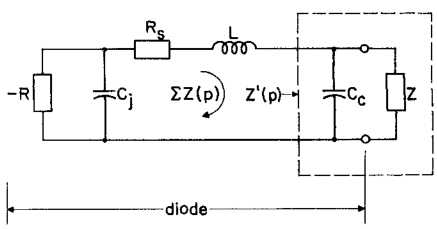

Fig. 1—Equivalent tunnel-diode amplifier representation showing loop chosen for stability criteria.

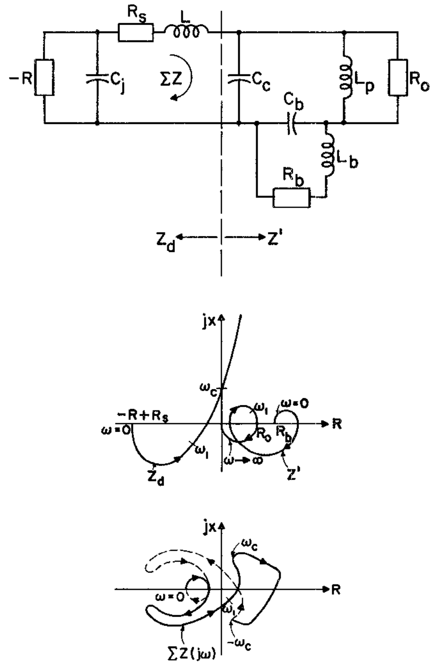

Fig. 2—Example of diode and connected network representating a stable amplifier.

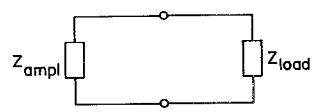

Fig. 3—Loop for determining permissible mismatch.

has no solutions in the right-half plane. If we define

$$G = (\text{ideal voltage gain}) = \frac{Z_{ampl} - R_0}{Z_{ampl} + R_0}$$

$$\rho = (\text{nonideal load reflection coefficient}) = \frac{Z_{load} - R_0}{Z_{load} + R_0},$$

then $Z_{ampl}+Z_{load}=0$ can be rewritten as $G\cdot\rho-1=0$.

As $G\cdot\rho$ has no poles in the right-half plane (G is stable and Z_{load} is a passive impedance), a necessary and sufficient stability criterion is that the complex plot of $G\cdot\rho$ when ω goes from $-\infty$ to $+\infty$ does not encircle the point $+1$. (Compare with the Nyquist criterion for feedback-amplifiers.)

If the phase of the input reflection coefficient is not known or not controlled, a sufficient criterion for stability is

$$|G|\cdot|\rho| < 1.$$

* Received March 26, 1962; revised manuscript received, May 4, 1962.

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[2] L. L. Smilen and D. C. Youla, "Stability criteria for tunnel diodes," PROC. IRE, vol. 49, pp. 1206–1207; July, 1961.

[3] S. Goldman, "Transformation Caculus and Electrical Transients," Prentice-Hall, Inc., New York, N. Y., pp. 370–371; 1950.

[4] W. L. Hughes, "Nonlinear Electrical Networks," Ronald Press Co., New York, N. Y.; pp. 166–168, 1960.

[5] U. S. Davidsohn, Y. C. Hwang and G. B. Ober, "Designing with tunnel diodes, part 1," *Elec. Design*, vol. 8, pp. 50–55; February, 3, 1960.